## **Chapter 2 Basic Circuit Connections and Laws**

**2.1 Circuit Terminology**

* + **Node:** junction of two or more circuit elements.
  + **Essential node:** junction of three or more circuit elements.
  + **Path:** set of one or more adjoining circuit elements that may be traversed in succession without passing through the same node more than once. A path generally has an initial node at its beginning and a final node at its end.
  + If the initial and final nodes are the same, the path is closed and becomes a **loop**.
  + **Mesh**: a loop that does not enclose any other loop.
  + **Branch:** path that connects two nodes.
  + **Essential branch:** branch that connects two essential nodes without passing through an essential node.
  + In Figure 2.1.1 the nodes labeled a, b, c, and d, are essential nodes, whereas nodes 1, 2, and 4 are not essential nodes. Nodes 3 and b are one and the same, as are nodes 3’, d, and 5 because no circuit element is connected between them, only a connection of zero resistance that is used for convenience of illustration.



* + *Rsrc*, *vSRC*, *L*1, *R*1, *L*4, *R*4, *C*3, and *R*3 taken individually, are branches. The combinations *Rsrc*-*vSRC,* *L*1-*R*1, *L*4-*R*4, and the individual branches *C*3 and *R*3are essential branches. The closed paths d-5-1-a-b-d and d-5-1-a-c-b-d are loops. The loops d-5-1-a-c-d, a-b-c-a, and d-c-b-d (going through *C*3) are meshes. *C*3 and *R*3 can also be considered to form a mesh.

**2.2 Kirchhoff’s Laws**

**Kirchhoff’s Current Law**

***Statement*** *At any instant of time, the sum of currents entering a node is equal to the sum of currents leaving the node*.

* At node N in Figure 2.2.1 KCL gives: *iA* + *iB* = *iC* + *iD*.
* KCL is a direct expression of conservation of current, which follows from conservation of charge, just as conservation of power follows from conservation of energy.



* Alternative statement of KCL: *at any instant of time, the algebraic sum of all the currents at any node is zero.*

*iA* + *iB*  where opposite signs are assigned to currents flowing towards the node and to currents flowing away from the node.



* KCL may be applied not just to a node but also to interconnected circuits or parts of a circuit.
* Figure 2.2.2a: equal and opposite currents flow in the two connections. In Figure 2.2.2b, *i* = 0.

**Kirchhoff’s Voltage Law**

***Statement*** *At any instant of time, the sum of voltage rises around any loop is equal to the sum of voltage drops around the loop.*



* *v*1 + *v*2 + *v*3 = *v*4 + *v*5 in Figure 2.2.3.
* Equivalent statement of KVL: *at any instant of time, the algebraic sum of the voltages around any loop is zero*, since voltage drops and voltage rises have opposite signs:

*v*1 + *v*2 + *v*3 – *v*4 – *v*5 = 0.

***Concept*** *KCL and KVL together are an expression of conservation of energy.*

* In taking a charge +*q* clockwise around the loop, the work done on the charge is *q*(*v*1 + *v*2 + *v*3). But the charge can do more work, *q*(*v*4 + *v*5), if (*v*1 + *v*2 + *v*3) < (*v*4 + *v*5), which violates conservation of energy. Taking the charge +*q* around a loop, which necessarily involves passing through nodes or essentials nodes between the circuit elements, implies that the charge +*q* is conserved at these nodes, that is, KCL applies.

**2.3 Voltage Division and Series Connection of Resistors**

***Definition*** *In a series connection of elements, the same current flows through all the elements.*

* In Figure 2.3.1a:



 (2.3.1)

* Applying Ohm’s law to the individual resistors:

 (2.3.2)

* If *Rm* denotes one of the resistors *R*1 to *Rn* and *vm* is the voltage drop across *Rm*, so that *vm* = *Rmi*, then dividing this relation by Equation 2.3.2:

, *m* = 1, 2, …., *n* (2.3.3)

* *vSRC* divides across the string of resistors in the ratio of each individual resistance to the total resistance. If *vSRC* = 6 V, *R*1 = 1 Ω, *R*2 = 2 Ω, and *R*3 = 3 Ω, the voltages across the resistors are *v*1 = 1 V, *v*2 = 2 V, and *v*3 = 3 V.
* It also follows that the voltages across any two resistors are in the ratio of the corresponding resistances and in the inverse ratio of the conductances:

 (2.3.4)

* Equation 2.3.2 may be expressed as:

*vSRC* (2.3.5)

* + An equivalent series resistor *Reqs* is defined such that if *vSRC* is applied across *Reqs*, the resulting current is *i* (Figure 2.3.1b):

 (2.3.6)

* + Comparing Equations (2.3.5) and (2.3.6), it follows that:

 (2.3.7)

* + In the case of the 1, 2, and 3 Ω resistors connected in series, *Reqs* = 6 Ω.
  + Because of the summation of positive quantities in Equation 2.3.7, *Reqs* is larger than the largest of the individual resistances.
  + If all the *n* resistances are equal, *Reqs = nR*.
* If the resistors in Figure 2.3.1a are represented by their conductances, then every resistance in Equation 2.3.7 is replaced by the reciprocal of its reciprocal conductance, which leaves the quantity unchanged. Thus:

 (2.3.8)

* + The conductances of the three series-connected 1, 2, and 3 Ω resistors are, respectively, 1, , and  S. The sum of their reciprocals is 6 S-1, and  S, which is of course the reciprocal of *Reqs*.

**Summary** In a series connection of resistors, the resistances, or the reciprocals of the conductances, add.



**2.4 Current Division and Parallel Connection of Resistors**

* ***Definition*** *In a parallel connection of elements, the same voltage is applied across all the elements.*
* In going around each mesh in Figure 2.4.1a, there is a voltage drop of *vSRC* and a voltage rise of *vSRC*, so that KVL is automatically satisfied.

KCL at node a or node b gives:

 (2.4.1)

* Applying Ohm’s law to the individual resistors, in terms of their conductances:

 (2.4.2)

* If *Gm* denotes one of the conductances *G*1 to *Gn*, and *im* is the current through *Gm*, so that *im* = *GmvSRC*, then dividing this relation by Equation 2.4.2:

, *m* = 1, 2, …., *n* (2.4.3)

* *i* divides between the paralleled resistors in the ratio of each individual conductance to the total conductance. Moreover, the currents through any two resistors are in the ratio of the corresponding conductances, or the inverse ratio of the resistances:

 (2.4.4)

* In the case of two resistors in parallel (Figure 2.4.2):



 (2.4.5)

 (2.4.6)

 (2.4.7)

If *vSRC* = 6 V, *R*1 = 1 Ω, and *R*2 = 2 Ω, then *i*1 = 6 A, *i*2 = 3 A, and *i* = 9 A.

* Equation 2.4.2 may be expressed as:

 (2.4.8)

* An equivalent parallel conductance *Geqp* is defined such *vSRC* applied between terminals ab produces the same current *i* (Figure 2.4.1b):

 (2.4.9)

* + Comparing Equations 2.4.8 and 2.4.9, it follows that:

 (2.4.10)

* + if three resistors of 1, 2, and 3 S are connected in parallel, *Geqp* = 6 S.
* If the resistors in Figure 2.4.1a are represented by their resistances, then every conductance in Equation 2.4.10 is replaced by the reciprocal of its reciprocal resistance, so that:

 (2.4.11)

* + The resistances of the three parallel-connected 1, 2, and 3 S resistors are,

respectively, 1, , and  Ω. The sum of their reciprocals is 6 Ω-1, and

 Ω, which is the reciprocal of *Geqp*.

* + If *Rm* is the smallest resistance in the parallel combination, it follows from Equation 2.4.11 that , so that . That is, the parallel resistance is smaller than the smallest resistance of the paralleled resistors.
  + If all the resistances are equal, *Reqp* = *R*/*n*.

**Summary** In a parallel connection of resistors, the conductances, or the reciprocals of the resistances, add.

* + - According to Equation 2.4.11, *Reqp* is the product of the *n* resistances divided by the sum of their products (*n* – 1) at a time. In the case of three resistors in parallel:

 (2.4.12)

For two resistors:

 (2.4.13)

**Example 2.4.1 Resistivity**

It is required to calculate the resistance *R* between opposite ends of a block of material of length *L* units and uniform cross-sectional area *A* square units (Figure 2.4.4).



***Solution*:** Let the block be divided into a number of unit cubes, each having a resistance *ρ* between opposite faces. A strip of length *L* units and cross-sectional area of one square unit will have *L* cubes in series and a resistance *ρL* end-to-end.

Since the block consists of *A* such strips in parallel:

 (2.4.14)

*ρ* is the **resistivity**, or **specific resistance**, of the material. It is an intrinsic property of the material and is independent of the size or shape of a given sample, just as density is independent of size and shape. According top Equation 2.4.14, the unit of resistivity is Ω-(unit length). For copper, *ρ* is approximately 1.7×10–6Ωcm at room temperature, or 1.7×10–8Ωm. The reciprocal of resistivity is **conductivity**, usually denoted by *σ*.

**Example 2.4.2 Series-Parallel Connection of Lamps**

100 lamps rated at 12 V, 6 W each are to be connected across a 240 V supply such that the rated voltage of 12 V is applied to each lamp. Design an appropriate arrangement of the lamps and calculate the equivalent resistance and the total current drawn from the supply.

***Solution*:** Since  = 20, then 20 lamps may be connected in series across the supply. The voltage across each lamp will be the rated voltage of 12 V, assuming the lamp resistances are all equal. To accommodate 100 lamps, 5 such series combinations will have to be paralleled across the supply. The resistance of each lamp is Ω at the normal operating temperature. The equivalent series resistance of 20 lamps is 24×20 = 480 Ω. The equivalent parallel resistance of five of these series combinations is  Ω. The lamp current at 12 V is  A, which is also the current in each series combination. The total current drawn from the supply is 5×0.5 = 2.5 A.

As a check, we may calculate *P*, the total power supplied. From the total current, *P* = 240×2.5 = 600 W. From the total resistance: . From the total number of lamps and their individual ratings: *P* = 100×6 W = 600 W.

* 1. **Δ-Y Transformation**
     + Given three resistances *Ra*, *Rb*, and *Rc* connected in Δ (Figure 2.5.1a), it is required to determine the resistances *R*1, *R*2, and *R*3 connected in Y (Figure 2.5.1b) that will make the two circuits equivalent between terminals a, b, and c. This means that the resistance seen between any two terminals is the same for the two circuits, the third terminal being connected in the same way in both cases.



* + - The inverse problem is also required: given three resistances *R*1, *R*2, and *R*3 connected in Y, it is required to determine *Ra*, *Rb*, and *Rc* of the equivalent Δ.
    - Equating the resistances between terminals a and b, with terminal c open:

 (2.5.1)

* Equating the resistances between terminals bc, with terminal a open:

 (2.5.2)

* Equating the resistances between terminals ac, with terminal b open:

 (2.5.3)

* Equations 2.5.1 to 2.5.3 are three independent equations. If *Ra*, *Rb* , and *Rc* of the Δ-circuit are given, these equations can be solved for *R*1, *R*2, and *R*3 of the equivalent Y-circuit:

 (2.5.4)

 (2.5.5)

 (2.5.6)

* Conversely, if the resistances *R*1, *R*2, and *R*3 of the Y-circuit are given, Equations 2.5.1 to 2.5.3 can be solved for *Ra*, *Rb* , and *Rc* of the equivalent Δ-circuit:

 (2.5.7)

 (2.5.8)

 (2.5.9)



* To help remember the above relations, the two equivalent circuits are superimposed (Figure 2.5.2). Each Y-resistance is the product of the Δ-resistances on either side of it divided by the sum of the three Δ-resistances. Conversely, each Δ-resistance is the product of Y-resistances two at a time divided by the Y-resistance that is opposite the given Δ-resistance.
* When the three resistances in either configuration are equal, the above relations reduce to:

*RΔ* = 3*RY*, and *RY* = *RΔ* /3 (2.5.10)

**Example 2.5.1 Δ-Y Transformation**

It is required to obtain the resistance seen between terminals cd in the circuit of Figure 2.5.4a.



***Solution*:** The lower abc Δ is transformed to its equivalent Y (Figure 2.5.4b), as in Figure 2.5.2.

This gives:  Ω,

 Ω, and  Ω. The 5 Ω resistance is then added to the 9 Ω to give 14 Ω, and the 7.5 Ω resistance is added to the 6.5 Ω to give 14 Ω. The two 14 Ω resistors in parallel give 7 Ω, which is added to the 3 Ω to give 10 Ω between terminals cd.

**2.6 Source Equivalence and Transformation**

* A practical voltage source departs from the ideal in that the voltage at its terminals decreases with increasing current supplied by the source. If the variation is linear, then at least in the case of dc sources, this effect may be simulated by adding a **source** **resistance** *Rsrc* in series with an ideal voltage source element (Figure 2.6.1).



* From KCL: *IL* = *ISRC* at nodes a or b. In going clockwise around the circuit, KVL gives:

 (2.6.1)

* Ohm’s law gives: . Substituting in Equation 2.6.1:

 (2.6.2)

* It is seen from Equation 2.6.2 that as *IL* increases, the voltage *VL* at the source terminals decreases. When *IL* = 0, *VL* = *VSRC*. Thus, *VSRC* is the **open-circuit voltage** of the source.
* When *IL* equals a specified, or rated, value, *VL* drops to the value given by Equation 2.6.2. This variation in *VL*, due to the loading effect of *IL*, is the **voltage regulation** of the voltage source and is equal, in percentage, to: .
* The corresponding case of a current source is shown in Figure 2.6.2. KCL at nodes a or b gives:



 (2.6.3)

* KVL is automatically satisfied since the same voltage *VL* is across all the circuit elements.
* Multiplying both sides of Equation 2.6.3 by *Rsrc* and substituting :

*VL* = *RsrcISRC* – *RsrcIL* (2.6.4)

* When *VL* = 0, *IL* = *ISRC*. As *VL* increases, more of *ISRC* is diverted to *Rsrc*, so *IL* decreases. **Current regulation** occurs, analogous to voltage regulation in a voltage source.
* Equations 2.6.2 and 2.6.4 are identical if *VSRC* = *RsrcISRC*, with *Rsrc* being the same in both cases. Under these conditions, if the two sources are hidden from view, and only the variation of *VL* with *IL* is observed, it would be impossible to tell which source is connected to *RL*. In other words, the two sources are *equivalent as far as the load is concerned*.



* It is concluded, therefore, that a voltage source *VSRC* in series with a resistance *Rsrc* is equivalent at its terminals to a current source  in parallel with the same resistance *Rsrc*. Conversely, a current source *ISRC* in parallel with a resistance *Rsrc* (or a conductance ) is equivalent at its terminals to a voltage source *VSRC* = *RSRCIsrc* in series with the same resistance *Rsrc*.
* Source transformation applies to dependent sources as well.
* By its very nature, source transformation preserves the polarities of voltage and current at the source terminals.
* The concept of equivalence of two circuits at a pair of terminals is of fundamental importance and may be generalized as follows:

***Concept*** *Two circuits are said to be equivalent at a specified pair of terminals if the voltage-current relation at these terminals is identical for both circuits.*

* It should be emphasized that the equivalence applies only at the specified terminals and does not apply, in general, to the rest of the circuit.

**Example 2.6.1 Equivalent Sources**

Given a voltage source having an open-circuit voltage of 12 V and a source resistance of 0.5 Ω (Figure 2.6.4a). It is required to derive the equivalent current source.



***Solution*:** According to the above discussion, the equivalent current source has a source current of  A and a source resistance of 0.5 Ω (Figure 2.6.4b).

If both sources are connected to a 5.5 Ω load, for example, *IL* for the voltage source case is, from Figure 2.6.4 (a): *IL*  =  A, and *VL* = 2×5.5 = 11 V. In the case of the current source (Figure 2.6.4b), the resistance of the parallel combination is  Ω; V, as in the case of the voltage source. However, the power delivered by the voltage source is  whereas the power delivered by the current source is 11×24 = 264 W. The power absorbed by *Rsrc* is  W in the case of the voltage source and  W in the case of the

current source. Moreover, the current in the 0.5 Ω resistor is not the same in both cases. It is 2A in (a) and 22 A in (b).

***Concept*** *The ideal termination for a voltage source is an open circuit, whereas the ideal termination for a current source is a short circuit.*

* This is because the full voltage *VSRC* is available at the terminals of a voltage source only when *IL* = 0, that is, when the source terminals are open circuited. Similarly, the full current *ISRC* is available at the terminals of a current source only when *VL* = 0, that is, when the source terminals are short circuited.

**2.7 Reduced-Voltage Supply**

* The simple resistive voltage divider is often used to supply a load at a reduced voltage.
* The basic circuit is shown in Figure 2.7.1. Normally, *VSRC* is given together with the nominal load voltage *VL* and a range of variation of the load current *IL*. It is desired to determine *R*1 and *R*2 such that the variation in *VL* remains within specified limits as *IL* changes over the given range.



* From KCL at either essential node:

 (2.7.1)

* Applying KVL clockwise around the mesh composed of *VSRC*, *R*1, and *R*2:

 (2.7.2)

* From Ohm’s law:

 and  (2.7.3)

* *I*1, *I*2 and *V*1 may be eliminated from the above four equations to give:

 (2.7.4)

 (2.7.5)

where is the parallel resistance of *R*1 and *R*2.



* Comparing Equation 2.7.5 with Equation 2.6.2, it

follows that the voltage divider appears to the load as a source of open-circuit voltage , when *IL* = 0, and source resistance  (Figure 2.7.2).

* Given *VSRC*, *VL*, and *IL*, Equation 2.7.5 provides a relation between *R*1 and *R*2, but is not sufficient for uniquely determining their values. The additional information needed for this purpose comes from specifying the voltage variation *ΔVL* = *VL*2 – *VL*1 for*ΔIL* = *IL*2 – *IL*1. Substituting first *VL*1 and *IL*1, in Equation 2.7.5, then *VL*2 and *IL*2, and subtracting, eliminates the term in *VSRC* and gives:

 (2.7.6)

* The interpretation of Equation 2.7.6 is that when  is constant and *IL* changes by *ΔIL*, *ΔVL* is due to the voltage drop in, which is ()*ΔIL*. The minus sign in Equation 2.7.6 signifies that *VL* decreases with *IL*.
* It is desirable in practice to have a small *ΔVL* for a given *ΔIL*, which means that should be small. It is seen from Equation 2.7.5 that this makes  small compared to the first term, , which means that . Under these conditions,  is fixed for given *VSRC* and *VL*.
* To reduce while keeping  constant, both *R*1 and *R*2 should be reduced. But reducing *R*1 and *R*2, increases the current drain from the supply, and generally increases the power dissipated in the voltage divider. This is undesirable, particularly in battery-operated equipment. Hence a tradeoff must be made in the design of the resistive voltage divider between good voltage regulation, that is, a small *ΔVL*, on the one hand, and current drain from the supply and power dissipation in *R*1 and *R*2 on the other hand. For these reasons, the resistive voltage divider is impractical where there is a substantial variation in load current.

**Design Example 2.7.1 Voltage Regulation and Power Dissipation in a Voltage Divider**

A 3 V, 0.1 A dc load is to be supplied from a 12 V dc supply. The load current may vary by 20% of its nominal value of 0.1 A. The corresponding variation in the load voltage should not exceed 5% of the nominal value of 3 V. It is required to determine suitable values of *R*1 and *R*2.

***Solution*:** According to Equation 2.7.5, *IL* has its largest value when *VL* has its smallest value, and conversely. At *IL* = 0.8×0.1 = 0.08 A, *VL* = 1.05×3 = 3.15 V, and at *IL* = 1.2×0.1 = 0.12 A, *VL* = 0.95×3 = 2.85 V. Substituting these values in Equation 2.7.5 gives two relations between *R*1 and *R*2:

, and 

From these, *R*1 = 24 Ω and *R*2 =10.9 Ω. Note that an easy way to solve these equations is to consider as one variable and as another variable.

Let us calculate *P*, the total power dissipated in *R*1 and *R*2 under these conditions. With reference to Figure 2.7.1, . When *VL* = 3.15 V, *P* = 4.17 W, and when *VL* = 2.85 V, *P* = 4.23 W. *P* may not seem like an excessive amount of power. But if it is noted that the power consumed by the load does not exceed 0.34 W, when *VL* = 2.85 V and *IL* = 0.12 A, it is seen that  of the power supplied by *VSRC* goes to the load under these conditions, the rest is dissipated in *R*1 and *R*2. The power efficiency is therefore quite poor.

Suppose that in order to dissipate less power in *R*1 and *R*2, *R*1 is increased by a factor of 3 to 72 Ω. From Equation 2.7.5: , which gives *R*2 = 120 Ω. With these values of *R*1 and *R*2, *VL* = 3 V when *IL* = 0.1 A, and the power dissipated in *R*1 and *R*2 is now W. From Equation 2.7.6, –1.8 V. Thus, as *IL* changes from 0.08 A to 0.12 A, *VL* changes from  V to  V, a variation of 30% of the nominal value of 3 V.

Such a variation will most likely not be acceptable.

If the load current is constant, as may occur in the case of a lamp or a heater, for example, *R*2 may be dispensed with altogether and only *R*1 used. If *IL* = 0.1 A and *VL* = 3 V, then  Ω and the power dissipated is  W. *R*1 is used in this case to drop the supply voltage by 9 V at 0.1 A.

**Design Example 2.7.2 Measurement of Partial Pressure of Oxygen**

The partial pressure of oxygen in the blood is measured as a direct indication of lung function, and as an indirect measurement of glucose concentration. Approximately 0.7 V is applied between two suitable electrodes, the resulting current being directly proportional to the partial pressure of oxygen in the solution. For the partial pressures normally of interest, the range of current is 0 to1.5 μA. It is required to determine suitable values of *R*1 and *R*2, assuming *VSRC* = 12 V and *VL* = 0.7 V.

***Solution*:** A good guide for selecting *R*1, according to Equation 2.7.4, is to have the largest value of *R*1*IL* small compared to *VSRC*, say one-tenth of *VSRC*. Then,  kΩ. The nearest smaller standard value of a 5% tolerance resistor is 750 kΩ. Using this value in Equation 2.7.5, with *VL* = 0.7 V and a mid-range value of *IL* = 0.75 μA, gives *R2* = 48.9 kΩ. The nearest smaller standard value of a 5% tolerance resistor is 47 kΩ. These values give *VL* = 0.708 V at *IL* = 0 and *VL* = 0.641 V at *IL* = 1.5 μA. The variation in *VL* is about 5 % with respect to the nominal value of 0.7 V, which is quite acceptable.

With *R*1||*R*2 = 44.23 kΩ, the power dissipated in *R*1 and *R*2 is  0.18 mW, which is small because *IL* is small, so that *R*1 and *R*2 are relatively large.